

ON THE PROBLEM OF OPTICAL BISTABILITY OF NONLINEAR COMPOSITES WITH COATED INCLUSIONS

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Abstract

The electrodynamic properties of the nonlinear metal composites are intensively studied in many papers [1-7]. One of the most important property of such systems is the abnormal enhancement of the nonlinear optical response in the composites containing small inclusions (compared to the wavelength of radiation) of a nonlinear dielectric covered by the metal shell embedded in a dielectric host matrix [2-6]. The surface plasmons in the metal shell may be tuned in resonance with the external electromagnetic field and produce a considerable increase in the local field in the core of inclusion to make the nonlinear part of its dielectric permittivity to be important. As a result, the connection between the applied and the local field in the core becomes nonlinear and in some diapason of applied electric fields even ambiguous. It happens to be that one value of the applied field corresponds to a few values of the local fields and polarization of the inclusions that in its turns leads to instability in the composite optical properties. This phenomenon is called the intrinsic optical bistability (IOB).

In this paper we calculate the dielectric permittivity and polarizability of a separate inclusion and analyze the parameters of the IOB. Further, we consider the dielectric function of

1. Local field in coated sphere particle

Let us consider a spherical inclusion covered with a metal shell of an outer radius r_2 . The core of the inclusion is the Kerr type nonlinear dielectric of a radius r_1 . We choose the dielectric function of the core in the form

$$\varepsilon_1 = \varepsilon_{10} + \chi |\vec{E}_1|^2 \quad (1.1)$$

where ε_{10} is a linear part of the dielectric function, χ is the nonlinear Kerr coefficient, and E_1 is an amplitude of the local field in the core. The dielectric function of the metal shell let be the Drude type

$$\varepsilon_2 = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega + i\nu)}, \quad (1.2)$$

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where ε_∞ does not depend on frequency ω ; ω_p , and ν are the plasma and collision frequency of conducting electrons respectively. The dielectric permittivity of the host matrix material we denote as ε_m .

Let the external varying with time electromagnetic field $\vec{E} = \vec{E}_0 e^{-i\omega t}$ acts on the inclusion. In

the long wavelength limit $r_2 \ll \langle \frac{\omega}{c} \sqrt{|\varepsilon_2|} \rangle$, $r_2 \ll \langle \frac{\omega}{c} \sqrt{|\varepsilon_m|} \rangle$ the distribution of the electric field within

the inclusion may be found by solving the Laplace equation for the two-layer sphere in the homogeneous constant electric field E_0 . By using the continuity conditions for the electric potential and the normal component of the electric induction on the inclusion interfaces one can get the local field as a function of the applied field

$$\vec{E}_1 = \gamma \vec{E}_0, \quad (1.3)$$

where γ is called the enhancement factor and given by the following expression

$$\gamma = \frac{9}{2p} \cdot \frac{\varepsilon_2^r}{\Delta}, \quad (1.4)$$

$$\Delta = (\varepsilon_2^r)^2 + [(3/2p - 1)\varepsilon_1^r + 3/p - 1]\varepsilon_2^r + \varepsilon_1^r$$

Here we use the relative dielectric functions with respect to ε_3 , in particular $\varepsilon_i^r = \varepsilon_i/\varepsilon_m$ ($i=1,2$), $p=1 - r_1^3/r_2^3$ is a fraction of the metal in the inclusion. Below, an index r will be omitted if it does not lead to confusion.

It is clear from (1.3) and (1.4) that one may obtain a considerable increase in the local electric field provided that $\Delta \rightarrow 0$. It can be done by tuning the parameters entering in (1.4). We consider the simplest case in the limit, $\nu/\omega \ll 1$, when an imaginary part of (1.2) is negligible. In this case the condition of the "resonance" corresponds to $\Delta=0$ that reduces to a quadratic equation in ε_2 (1.4). Roots of this equation may be written in the form

$$\varepsilon_{2\pm} = (-\beta \pm \sqrt{\beta^2 - 4\varepsilon_1^r})/2, \quad \beta = (3/2p - 1)\varepsilon_1^r + 3/p - 1. \quad (1.5)$$

We note that in our case always $\beta < 0$ and the roots $\varepsilon_{2\pm} < 0$. That is why $\varepsilon_{2-} < \varepsilon_{2+}$ and as well. In the limiting case of small metal fraction, $p \ll 1$, expressions (1.5) may be simplified:

$$\varepsilon_{2-} = -\frac{3\varepsilon_1^r + 2}{2p}, \quad \varepsilon_{2+} = -\frac{2\varepsilon_1^r}{3\varepsilon_1^r + 2} p, \quad p \ll 1 \quad (1.6)$$

Since $\varepsilon_{2\pm}$ are negative the realization of the enhancement of the local field requires that $\text{Re} \varepsilon_2(\omega)$ will be negative as well. One can see from (1.2) that it takes place at frequencies $\omega < \omega_0 = \omega_p / \sqrt{\varepsilon_\infty}$ (we assume that $\omega_0 \gg \nu$). The frequency ω_0 corresponds to the frequency of bulk plasmons. It also follows from (1.2) that a maximum enhancement of the local field occurs when the frequency of electromagnetic wave approaches to

$$\omega_{\pm\pm} = \frac{\omega_p}{\sqrt{\varepsilon_\infty - \varepsilon_{2\pm}}}, \quad (1.7)$$

For example, at $p \ll 1$, when the metal fraction of inclusion is comparatively small, $\omega_{s-} \rightarrow 0$, $\omega_{s+} \rightarrow \omega_0$.

The enhancement of local field in the inclusion core means that the nonlinear term in (1.1) must be taken into account. Therefore, the enhancement factor γ (1.4) itself depends on the local field E_1 and, in a general case, relation (1.3) turns out to be the equation for the local field as a function of applied field E_0 . Inserting (1.1) into (1.4) and multiplying a new expression (1.3) by its complex conjugated, we obtain the following cubic equation for $\bar{E}_1 \cdot \bar{E}_1^* = |\bar{E}_1|^2$

$$\begin{aligned} x^3 + 2 \operatorname{Re}\left(\frac{\Delta_0}{\delta}\right)x^2 + \left|\frac{\Delta_0}{\delta}\right|^2 x &= \eta x_0, \\ x = \chi^r |\bar{E}_1|^2 > 0, \quad x_0 = \chi^r |\bar{E}_0|^2 > 0, \quad \Delta_0 = \Delta(\varepsilon_1^r \rightarrow \varepsilon_{10}^r), \\ \delta = 1 + \varepsilon_2^r (3/2p - 1), \quad \eta = \frac{81}{4p^2} \left|\frac{\varepsilon_2^r}{\delta}\right|^2, \quad \chi^r = \chi^r / \varepsilon_m. \end{aligned} \quad (1.8)$$

This cubic equation has real coefficients and may have one real positive root or three real positive roots depending on its parameters. It will be analyzed in detail in the next section. Now we would like to pay more attention to the physical meaning of this result. Appearance of three different values of the local field that correspond to one value of the applied electric field means that the system becomes unstable. This phenomenon is called the intrinsic optical bistability (IOB) [6] and associated with a sudden change in the optical properties of the inclusion and the disperse system as a whole depending on the amplitude of the incident radiation.

Here we would like to note that the polarizability of the two-layer spherical inclusion may be presented in the following form [11]

$$\alpha = 4\pi r^3 \frac{\bar{\varepsilon} - \varepsilon_m}{\bar{\varepsilon} + 2\varepsilon_m}, \quad (1.9)$$

where $\bar{\varepsilon}$ is the effective dielectric function of the individual two-layer inclusion in the dipole approximation and given by the relation

$$\bar{\varepsilon} = \varepsilon_2 \frac{\varepsilon_1(3/p - 2) + 2\varepsilon_2}{\varepsilon_1 + \varepsilon_2(3/p - 1)} \quad (1.10)$$

A detail study of scattering and absorbing properties of such particles in the linear approximation (with respect to the electric field) when there is no nonlinear term in ε_1 (1.1) is given in [12]. Combining (1.10) and (1.9) one may easily show that the enhancement factor (1.4) and the polarizability coefficient (1.9) have the same denominator Δ (1.4). This means that α considerably increases when the frequency ω approaches to one of the frequencies (1.7). At the same time, the absorption of radiation by the particle increases due to increasing in the polarization.

2. Optical bistability in coated sphere particle

Let us rewrite the cubic equation (1.8) in the following form

$$\begin{aligned} x^3 + ax^2 + bx + c &= 0, \\ a = 2 \operatorname{Re}\left(\frac{\Delta_0}{\delta}\right), \quad b = \left|\frac{\Delta_0}{\delta}\right|^2, \quad c = -\frac{81}{4p^2} \left|\frac{\varepsilon_2^r}{\delta}\right|^2 x_0. \end{aligned} \quad (2.1)$$

In a general case the location of roots of equation (2.1) on a complex plane depending on the coefficients a, b, c is given in Table 1.

Table 1 Appendix

Range of parameters	Location of roots on the complex plane
$x^3 + ax^2 + bx + c = 0, Q = (H/3)^3 + (G/2)^2 < 0,$	
$H = -a^2/3 + b, G = 2(a/3)^3 - ab/3 + c$	
in this case all roots are real	
$ab - c < 0,$ $c < 0, b > 0$ (1)	
$ab - c > 0,$ $c > 0, b > 0$ (2)	
$ab - c < 0,$ $c > 0, b > 0$ or $c > 0, b \leq 0.$ (3)	
$ab - c > 0,$ $c < 0, b > 0$ or $c < 0, b \leq 0.$ (4)	
$x^3 - ax^2 + bx + c = 0, Q = (H/3)^3 + (G/2)^2 > 0,$	
$H = -a^2/3 + b, G = 2(a/3)^3 - ab/3 + c$	
in this case one root is real and two roots are complex conjugated	
$ab - c < 0,$ $c < 0, b > 0.$ (5)	
$ab - c > 0,$ $c > 0, b > 0.$ (6)	
$ab - c < 0,$ $c > 0, b > 0$ or $c > 0, b > 0.$ (7)	
$ab - c > 0,$ $c < 0, b > 0$ or $c < 0, b \leq 0.$ (8)	

In our case $b > 0$, $c < 0$. Therefore, depending on a sign of the discriminant

$$Q = (b/3 - a^2/9)^3 + (a^3/27 - ab/6 + c/2)^2 \quad (2.2)$$

equation (2.1) has one (point 5 of the Appendix) or three (point 1 of the Appendix) real positive roots. We may state that the two-layer particle manifests the IOB provided that

$$Q < 0, \quad ab - c < 0. \quad (2.2a)$$

These inequalities in principle allow one to find the borers of IOB.

However, this system of inequalities is not convenient for a physical analysis because of the high powers of the coefficients a , b , c . Now we use some geometrical considerations on the real plane x , y . In our particular case, three positive roots of equation (2.1) emerge if a graph of the function $f(x) = x^3 + ax^2 + bx + c$ at real positive x crosses three times a straight line $y(x) = -c$. (See Fig. 1). It happens if a derivative $f'(x)$ turns out to be zero at two different points x_1 and x_2 ($0 < x_1 < x_2$) and the straight line lies between $f(x_1)$ and $f(x_2)$. At the same time, $f(x_1) > f(x_2)$ which means that $f'(x) < 0$ in the interval (x_1, x_2) . These conditions may be reduced to the following system of inequalities

$$\begin{aligned} a &< -\sqrt{3b}, \\ f(x_2) &< c < f(x_1), \end{aligned} \quad (2.3)$$

where $x_{1,2} = (-a \mp \sqrt{a^2 - 3b})/3$ are the roots of the quadratic equation $f'(x) = 0$.

In our notations (2.3) takes the form

$$\operatorname{Re}(\Delta_0 / \delta) < -\sqrt{3} \operatorname{Im}(\Delta_0 / \delta),$$

$$f(x_2) < \frac{81}{4p^2} \left| \frac{\varepsilon_2'}{\delta} \right| x_0 < f(x_1), \quad (2.4)$$

$$x_{1,2} = \{-2 \operatorname{Re}(\Delta_0 / \delta) \mp \sqrt{[\operatorname{Re}(\Delta_0 / \delta)]^2 - 3[\operatorname{Im}(\Delta_0 / \delta)]^2}\} / 3,$$

$$\Delta_0 = (\varepsilon_2')^2 + \varepsilon_2' [\varepsilon_{10}' (3/2p - 1) + 3/p - 1] + \varepsilon_{10}', \quad \delta = 1 + \varepsilon_2' (3/2p - 1).$$

Since in our case $\varepsilon_{10} > 0$ and $\varepsilon_3 > 0$, there is no IOB in the frequency region where $\operatorname{Re} \varepsilon_2(\omega) > 0$. The second inequality of (2.4) determines the range of the applied field where IOB appears.

Now we again come back to the simplified version of the theory when the imaginary part of ε_1 is negligibly small. In this situation the first condition of the IOB reduces to

$$\Delta_0 \delta < 0 \quad (2.4)$$

Using the results of the first section, we may present it in the following way

$$(\omega - \omega_{s-}^{(0)}) (\omega - \omega_{s+}^{(0)}) (\omega - \omega_{s1}) < 0 \quad (2.5)$$

where the frequencies $\omega^{(0)}$, are given by formula (1.7) where ε_1 is substituted with ε_{10} and

$$\omega_{s1} = \frac{\omega_p}{\sqrt{\varepsilon_\infty - \varepsilon_0}}, \quad \varepsilon_0 = 1/(3/2p - 1) \quad (2.6)$$

It is clear that $\omega_{s-}^{(0)} < \omega_{s+}^{(0)} < \omega_{s1}^{(0)}$. In addition in this case

$$x_1 = -\Delta_0 / \delta, \quad x_2 = -3\Delta_0 / \delta,$$

$$f(x_1) = -\frac{4}{27} \left(\frac{\Delta_0}{\delta}\right)^3, \quad f(x_2) = 0 \quad (2.7)$$

Summing up these results, we can state that the IOB emerges in the two-layer spherical particles with the Kerr-like nonlinear dielectric core (dielectric function (1.1)) covered with the metal shell with no decay (the Drude type dielectric function (1.2) with zero imaginary part) in the following frequency bands:

$$0 < \omega < \omega^{(0)}_{s_2} \quad \text{and} \quad \omega^{(0)}_{s_1} < \omega < \omega^{(0)}_{s_1} \quad (2.8)$$

The IOB does not exist above the external applied fields that exceed the critical value

$$E_c = \frac{4\rho}{27|\epsilon_2^r|} \sqrt{\frac{|\Delta_0|^3}{3\chi^r \delta}} \quad (2.9)$$

Influence of a small imaginary part of ϵ_2 can be performed by using an expansion with respect to the small parameter and slightly changes the obtained results. The case with finite decaying requires numerical methods and will be done in the next section.

3. Numerical calculations of IOB in coated spherical particle

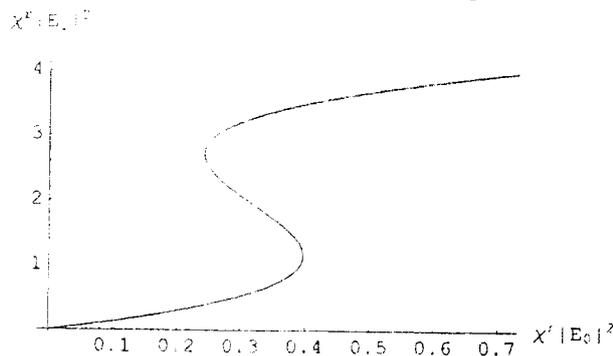


Fig. 1. $p=0.2$, $\epsilon_m=1.5$, $\epsilon_{inf}=1$, $\Omega=\omega/\omega_p=0.1$, $\Gamma=0.01$, $\epsilon_{r10}=5$.

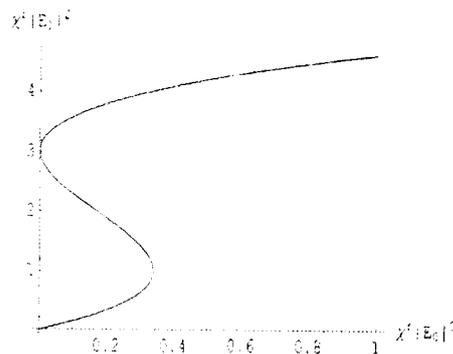


Fig. 2. $p=0.2$, $\epsilon_m=1.5$, $\epsilon_{inf}=1$, $\Omega=0.1$, $\Gamma=0$, $\epsilon_{r10}=5$.

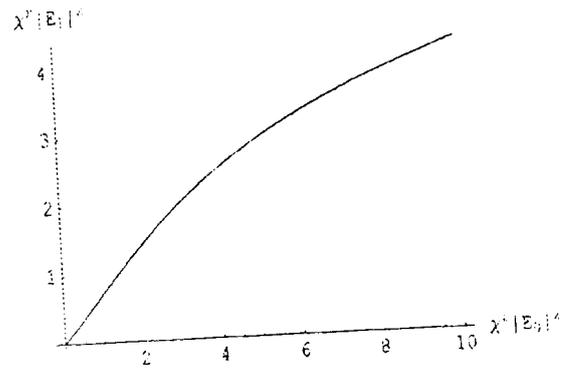


Fig. 3. $p=0.2$, $\epsilon_m=1.5$, $\epsilon_{inf}=1$, $\Omega=0.1$, $\Gamma=0.05$, $\epsilon_{r10}=5$.

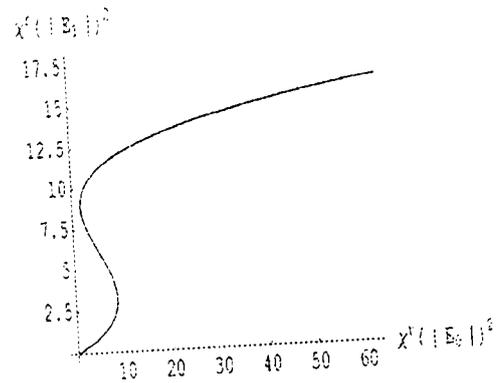


Fig. 4. $p=0.3$, $\epsilon_m=1.5$, $\epsilon_{inf}=1$, $\Omega=0.1$, $\Gamma=0.01$, $\epsilon_{r10}=5$.

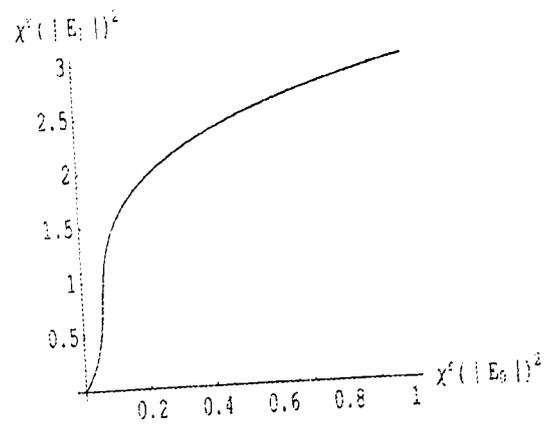


Fig. 5. $p=0.17$, $\epsilon_m=1.5$, $\epsilon_{inf}=1$, $\Omega=0.1$, $\Gamma=0.01$, $\epsilon_{r10}=5$.

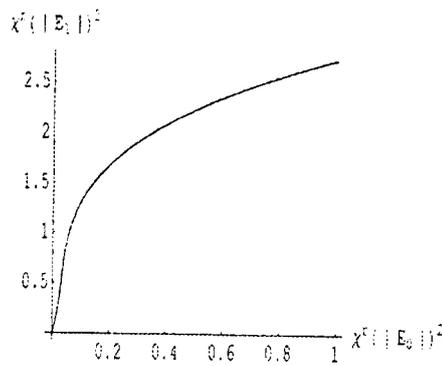


Fig. 6. $p=0.2$, $\epsilon_m=1.9$, $\sin f=1$, $\Omega=0.1$, $\Gamma=0.01$, $\epsilon_{r10}=5$.

4. Effective dielectric permittivity of nonlinear composite with two-layer inclusions

While studying the processes of interaction of electromagnetic radiation and matrix disperse systems (MDS) the method of effective medium is widely used. Usually, a nonmagnetic MDS with discretely distributed inclusions (the dielectric permittivity $\bar{\epsilon}$) embedded a continuous host matrix (the dielectric permittivity ϵ_m) is substituted with a continuous media that described by the effective dielectric permittivity $\tilde{\epsilon}$ depending on $\bar{\epsilon}$, ϵ_m , a density number of inclusions n , and their statistical distribution in the host matrix. This method works especially good in the long wave length approximation. Below we use two the most reliable approaches for calculation $\tilde{\epsilon}$.

At small density numbers of inclusions when their relative fraction $f = \frac{4}{3}\pi r_2^2 n$ is small, $f \ll 1$, the Maxwell-Garnet (MG) approximation is usually used [9]

$$\frac{\tilde{\epsilon} - \epsilon_m}{\tilde{\epsilon} + 2\epsilon_m} = f \frac{\bar{\epsilon} - \epsilon_m}{\bar{\epsilon} + 2\epsilon_m} \quad (1.3)$$

At higher density numbers of inclusions the Bruggeman approximation is more relevant [10]

$$f \frac{\tilde{\epsilon} - \bar{\epsilon}}{\bar{\epsilon} + 2\tilde{\epsilon}} + (1-f) \frac{\tilde{\epsilon} - \epsilon_m}{\tilde{\epsilon} + 2\epsilon_m} = 0 \quad (2.3)$$

Here $\bar{\epsilon}$ is given by relation (1.10). In this relation we have to insert $\epsilon_1 = \epsilon_{10} + \chi|\bar{E}_1|^2$ after obtaining the local field (or more precisely $\chi|\bar{E}_1|^2$) from equation (2.1).

Now we consider the case of small amplitudes of the external field E_0 when $\chi|\bar{E}_1|^2 \ll \epsilon_{10}$. Expanding (1.10) with respect to small parameter, we may obtain with the same accuracy the effective dielectric function of the inclusion particle

$$\bar{\epsilon} = \bar{\epsilon}_0 + \bar{\chi} |\bar{E}_1|^2, \quad (3.3)$$

where

$$\bar{\epsilon}_0 = \epsilon_2 \frac{\epsilon_{i0}(3/p-2) + 2\epsilon_2}{\epsilon_{i0} + \epsilon_2(3/p-1)}, \quad \bar{\chi} = \chi \frac{9(1/p-1)\epsilon_2^2}{[\epsilon_{i0} + \epsilon_2(3/p-1)]^2} \quad (4.3)$$

While calculating the nonlinear effective dielectric permittivity we follow the scheme suggested in [11]. In this paper, it is shown that for a system consisting of N nonlinear components with dielectric functions $\epsilon_i = \bar{\epsilon}_{oi} + \bar{\chi}_i |\bar{E}_i|^2$ ($i=1, \dots, N$) and respective fractions f_i the effective nonlinear dielectric function is given by the relation

$$\tilde{\epsilon} = \tilde{\epsilon}_0 + \sum_{i=1}^N \frac{\chi_i F_i |F_i|}{f_i} |\bar{E}_0|^2, \quad (5.3)$$

where $F_i = \frac{\partial \tilde{\epsilon}_0}{\partial \epsilon_{i0}}$ and $\tilde{\epsilon}_0$ is given by the generalized formulas (1.3) or (2.3)

$$\frac{\tilde{\epsilon}_0 - \epsilon_m}{\tilde{\epsilon}_0 + 2\epsilon_m} = \sum_{i=1}^N f_i \frac{\epsilon_{i0} - \epsilon_m}{\epsilon_{i0} + 2\epsilon_m}, \quad (6.3)$$

$$\sum_{i=1}^{N+1} f_i \frac{\epsilon_{i0} - \tilde{\epsilon}_0}{\epsilon_{i0} + 2\tilde{\epsilon}_0} = 0. \quad (7.3)$$

In the case of generalized MG and Bruggeman formulas (6.3) and (7.3) we obtain

$$F_i = \frac{9f_i \epsilon_m}{(\epsilon_{i0} + 2\epsilon_m)^2 (1 - \sum_{i=1}^N f_i \frac{\epsilon_{i0} - \epsilon_m}{\epsilon_{i0} + 2\epsilon_m})^2}, \quad (8.3)$$

$$F_i' = \frac{f_i \tilde{\epsilon}_0}{(\epsilon_{i0} + 2\tilde{\epsilon}_0)^2 \cdot \sum_{i=1}^{N+1} \frac{f_i \epsilon_{i0}}{(\epsilon_{i0} + 2\tilde{\epsilon}_0)^2}} \quad (9.3)$$

Setting $N=1$ in these formulas we get

$$F_1 = \frac{9f\epsilon_m^2}{[\bar{\epsilon}_0 + 2\epsilon_m - f(\bar{\epsilon}_0 - \epsilon_m)]^2}, \quad F_1' = \frac{\tilde{\epsilon}_0 f}{\bar{\epsilon}_0 f + \beta(1-f)\epsilon_m}, \quad (10.3)$$

where $\beta = \left(\frac{\bar{\epsilon}_0 + 2\tilde{\epsilon}_0}{\epsilon_m + 2\tilde{\epsilon}_0}\right)^2$.

For small density numbers of inclusions, $f \ll 1$, in the linear approximation with respect to this small parameter, both formulas (10.3) give the same result

$$F_1 \approx \left(\frac{3}{1 + 2\tilde{\epsilon}_0 / \epsilon_m}\right)^2, \quad (11.3)$$

Inserting this into (5.3), we find the expression for the effective nonlinear dielectric permittivity

$$\tilde{\epsilon} = \tilde{\epsilon}_0 + \bar{\chi} |\bar{E}_0|^2, \quad (12.3)$$

where

$$\bar{\epsilon}_0 = \epsilon_m \left[1 + 3f \frac{\bar{\epsilon}_0 - \epsilon_m}{\bar{\epsilon}_0 + 2\epsilon_m} \right], \quad \bar{\epsilon}_0 = \epsilon_2 \frac{\epsilon_{10}(3/p-2) + 2\epsilon_2}{\epsilon_{10} + \epsilon_2(3/p-1)},$$

$$\tilde{\chi} = \chi \left(\frac{3}{2 + \bar{\epsilon}_0 / \epsilon_m} \right)^2 \cdot \left| \frac{3}{2 + \bar{\epsilon}_0 / \epsilon_m} \right|^2 \cdot \frac{9f(1/p-1)\epsilon_2^2}{[\epsilon_{10} + \epsilon_2(3/p-1)]} \quad (13.3)$$

Formulas (12.3) and (13.3) enables us to find the effective nonlinear dielectric permittivity of the composite using the known dielectric permittivities of the inclusion core ϵ_1 , the metal shell ϵ_2 , and the matrix material ϵ_m provided that $\chi |\bar{E}_1|^2 \ll \epsilon_{10}$ and $f \ll 1$.

Now we note that the expression for $\tilde{\chi}$ may be written in another form by using the results of Section 1:

$$\tilde{\chi} = \chi \cdot f(1-p) \frac{729\epsilon_m^4 \epsilon_2^2 |\epsilon_{10} p + \epsilon_2(3-p)|^2}{16p^4 [(\epsilon_2 - \epsilon_{2-})(\epsilon_2 - \epsilon_{2+})]^2 [(\epsilon_2 - \epsilon_{2-})(\epsilon_2 - \epsilon_{2+})]^2} \quad (14.3)$$

From this relation one can see that at frequencies of the incident electromagnetic radiation close to the frequencies of surface plasmons and provided that the imaginary part of ϵ_2 is comparatively small, a considerable enhancement of the effective Kerr coefficient $\tilde{\chi}$ may occur. The numerical evaluations show that for inclusions with silver covering a ratio $\tilde{\chi}/\chi$ may be of the order of 10^5 - 10^6 .

Conclusions

The perspectives of usage of the nonlinear metal composites (NMC) in science and engineering have been discussed partly in [1-5]. In these papers, some types of nonlinear dielectrics were named as the most promising for a further study.

Here we would like to stop on some theoretical conclusions that follow from the reported study. From the results obtained in Section 1 we can see that an enhancement of the local field in MDS with two-layer inclusions is possible only provided that a real part of the dielectric function of a covering must be negative. For the dielectric functions of Drude's type it takes place at frequencies $\omega < \omega_p / \sqrt{\epsilon_\infty}$. It is known that in this range of frequencies the weakly decaying surface modes exist in metals. Therefore, the metal covering of the nonlinear dielectric makes easier an enhancement of the local field to the required level.

One more important fact is also worth noting. For the following NMC structures the local field may be found by solving a cubic equation: a two-layer plane structure, a sphere of nonlinear dielectric in a metal host matrix and a two-layer ellipsoid of a nonlinear dielectric with a metal covering provided that the external electric field is parallel to one of its axis. It is typical that for the Kerr type of nonlinear dielectric the coefficient $b > 0$ and the coefficient $c < 0$. However, the order of the corresponding equation is equal to five for a spherical anisotropic dielectric inclusion with a metal core. For the two-layer ellipsoid that is similar to our sphere in the structure at arbitrary orientation of the external electric field with respect to the ellipsoid axes the order of this equation becomes seven. These changes in the order of equations may lead to the richer picture of the IOB in these systems. Moreover, taking into account the damping of electromagnetic radiation may considerably change the conditions of the IOB and a magnitude of the enhancement coefficient.

Acknowledgement

The authors acknowledge financial support from the National Science Foundation through the Faculty Early Career Development (CAREER) Award ECS-9624486 and an Easter Europe Program Supplement.

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